Design and Presentation of an Approach to Improve the Tehran Metro Comprehensive Traffic System and Retrieve Timetables in the Disturbance Conditions Using a Robust Optimization Method

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The Tehran Metro is usually subject to undesirable disruptions that can cause minor, major and sometimes unavoidable deviations from the original timetables. Based on the principles of disruption management, the Tehran Metro network solutions eliminate distortions such as train delays, dispatch cancellations, and passenger congestion through an intelligence-comprehensive traffic system. Considering the size and scope of Tehran metro lines, a computational solution according to the dimensions of problem-solving approaches is required to achieve optimal decision making. One model presented in this research is an event-activity network to improve the comprehensive metro traffic management software. The research results show that the establishment of optimization models in the comprehensive metro system increases the success rate of operational results and the probability of traffic controller's decisions in the justified set of problem solutions. Thus, robust optimization methods can meaningfully improve the overall productivity index of subway lines and expand the capacity level of the subway network. In the present research, an attempt was made to estimate the ratio of the active trains on the line sections and the maximum possible number of trains. Especially during peak hours and taking into account the innovative structures. The capacity of trains in Tehran Metro's comprehensive system for creating the timetable is increased. In case of disruptions, it returns to normal conditions. Therefore, in the face of complications caused by multiple passenger dispatches, a neighborhood search algorithm was used to analyze passenger demand and to improve system.

KEYWORDS

- ~ Tehran metro
- ~ Robustness
- ~ Optimization approach
- ~ Comprehensive system

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1. INTRODUCTION

Metro is one of the most important means of urban transportation and plays a key role in transporting citizens, especially in metropolitan areas (Aydin and Tirkolaee, 2022). Metro systems are a simple and interactive collection of all technical and social facilities and equipment that form the core of citizens' movement patterns and ensure their transportation in the vital and crowded parts of the city (Zhou et al., 2020). A crucial aspect of the metro transportation system is the management of train capacity and traffic flow on the railroad lines. This demonstrates the ability of the metro network to handle the number of passengers and traffic for each train in each hour and direction. This is assessed using various calculation methods (Seyedvakili et al., 2018). When managing the metro network, various traffic parameters must be considered to address train dispatching and acceptance problems. An important factor in determining train lines and fleet capacity, which is influenced by several factors. Identifying and verifying this capacity can enhance operational efficiency (Zeng et al, 2021). Some of the most important factors affecting the timetabling process include route capacity, railroad infrastructure, technical characteristics of the train fleet, signaling, control, and the original operating plan (Dolinayova et al, 2020). Currently, due to the diversity of train fleets (DC, AC, 300 and 500 series) and railroad lines (metro, overground subway, multiple gradients or horizontal) in the Tehran metro network, the adjustment of schedules and calculation of line capacity is currently mainly based on the UIC 405 relationship, (Qiang et al., 2014). The implementation of comprehensive metro scheduling systems can provide real-time accurate data on the current status of the train fleet and the condition of the rail lines. This is a basic capability to check their impact on timetables when performing calculations(Laporte et al., 2014). In other words, the primary objective of the present study is to improve the train schedule and manage the capacity of the metro network. This includes providing strategies to improve the headway of the lines, by considering buffer times into account. The Tehran metro network has seven active lines and two branches (lines one and four). fixed and mobile facilities and equipment of comprehensive transportation systems, such as computer-aided traffic analysis software, can provide a suitable tool for collecting, storing and processing data for schedule design, including a basic database. in addition, the Tehran metro comprehensive timetable system (TCS) can provide comprehensive data management for the metro network infrastructure to ensure the timely update of traffic information and its impact on schedules (Marín and García-Ródenas, 2009). the connections between stations are called edges. the metro network is established by the sum of edges, nodes, arcs connecting edges and nodes, control equipment, and rail facilities. Figure 1 shows the Tehran metro network with its nodes and edges.



Figure 1. The set of nodes and edges of the Tehran metro network (source: Tehran Metro technical document archive)



The time span of a moving train in the metro network can be represented using the headway [0, T], which indicates the headway. This interval can be expressed as a series of numbers, e.g. t = 0, 1, 2, 3. Takeover Metro trains begin their passenger journeys at the departure station and complete a round trip, returning to the same station after stopping at various stations along the route. The train rotation diagram for the Tehran subway is displayed in Figure 2.



Figure 2. Train circulation diagram of the Tehran Metro (source: Tehran subway technical documents archive)

The Tehran Metro carries more than two million passengers each day. An analysis of Tehran Metro traffic statistics shows that there are an average of ten disruptions per day, ranging from minor to major disruptions. This makes it difficult to accurately implement pre-planned schedules. Finding effective solutions to optimize headway and restore timetables after a disruption in the metro network can greatly enhance operational efficiency (Manríquez et al, 2020). Rescheduling timetables during traffic disruptions is a complex and time-consuming process from a computational perspective (Zhang et al., 2018). Researchers have primarily focused on the planning stages for train passage and movement in the event-activity approach. The increasing computational complexity of schedule recovery models has impacted the rerouting problem, making the convergence between the functional results of the model and the feasibility of implementation very important.

2. LITERATURE REVIEW

An extensive number of studies have been done related to the subject area of this research. Researchers have attempted to manage potential disruptions in the metro network by collecting and analyzing traffic data to restore schedules and return to normal conditions. one of these studies is by Marín and Perea (2013). The previous study aimed to maintain the overall performance of subway systems in optimal conditions, even during unpredictable disturbances. In the research conducted by Canca et al. (2020), the researchers proposed a computational model for the planning and designing mass rapid inter-city rail transportation using an adaptive local search algorithm for integrated rail networks. The model presented in this study aimed to integrate the fundamental processes of rail network planning addressing both route planning and fleet planning on a small scale for Seville city, Spain. The University of Wisconsin conducted studies on metro traffic management systems to analyze schedules and capacity management, which serve as the basis for the current research. These studies confirmed the factors that influence the stability of schedules and how infrastructure elements, including routes and fleet, impact the capacity management of metro lines. In a study conducted by Wang et al. (2022), the researchers allocated various resources to improve the rail network. They investigated a multi-objective optimization problem related to passenger flow in the Beijing subway aiming to minimize the operational costs of trains and reduce passenger waiting times. This was done while considering constraints related to the distribution of passengers and running trains across 16 lines. The study's findings revealed that by reducing the headway on lines 4 and 10 of the Beijing subway, the network's reliability indicator improved from 60 to 70. Additionally, passenger waiting times decreased by one percent. In their research, Wang and Fang (2017) focused on reducing unplanned delays in the operation and movement of trains within the fleet. Their approach involves rescheduling the timetable to achieve this goal.

In the Tehran metro network, resources such as platforms, signaling systems and track blocks are allocated to trains based on a timetable. The Tehran metro network consists of 217 trains transporting an average of 2 million passengers daily. Consequently, the number of train departures in the Tehran metro is quite high. Unexpected events such as overcrowding, equipment malfunctions or bad weather can disrupt train operations. While trains typically run on schedule, these unforeseen issues can affect service. Conflicts with other trains can complicate the situation, making it difficult to implement corrective measures and restore the normal timetable (Xu et al., 2016). The embedded database in the comprehensive traffic system, which relies on the insights of traffic experts, lacks the required comprehensiveness. In addition, the instructions given to traffic control center operators for explaining and identifying various aspects are sometimes contradictory. Activities related to the schedule redesign for managing disturbances in metro networks are generally divided into three categories: Rescheduling, stable sorting, and rerouting the fleet to resolve conflicts and minimize delays. Additional dispatching is the final step that allows traffic controllers to add an auxiliary train from the shunting and terminal areas. So we can consider passenger waiting time at each station as a key indicator of passenger demand. This information can assist in dispatching the trains and implementing strategies to reduce congestion, including optimizing the technical components of the trains (Jia et al., 2021). Recently, metro incident management systems have adopted a specifically passenger-oriented



approach, focusing on network topology and problem structure (Dmitrievskii et al., 2022). Through the above details, some of the innovations of the present research are listed as follows:

- The ability to calculate and manage passenger demand based on the metro network capacity, utilizing real-time data from transportation systems and information systems connected to rail components, train fleets and stations.
- The probability of effectively managing traffic disruptions and enhancing reliability of metro lines as we return to normality.
- Investigating a new method for calculating metro line capacity based on fluctuating passenger demand.

3. COMPREHENSIVE TRANSPORTATION SYSTEM OF THE TEHRAN METRO

The need to monitor and control trains on the rails and deal with the possibility of disruptions prompted the Tehran Metro Company to automate trip and movement planning. This was accomplished by the design of an application known as a comprehensive timetable system. The system uses reliable data from signaling systems to detect trains on the rail network. This system tracks train schedules and movements using timetables. It can also identify disruptions, calculate each train capacity at each station and estimate demand based on this capacity. The software consists of graphical symbols that help traffic operators allocate the metro network capacity according to different demands. Furthermore, the system includes equations for capacity enhancement and solutions to address traffic disturbances in a web environment. It also includes tools for mining specific data, alongside the general monitoring of railway lines and components:

- The technical specifications of each train include its weight, acceleration, braking force, the arrangement of wagons based on motorization and the axle load of each bogie.
- We also need to know the number of active trains on the line, the speed of the trains on the line sections, the train number and the departure and arrival times of each train in the service.
- Lastly, we need to determine the travel time of each train on the railway track.

The Tehran Metro timetable software is essential for planning train movements, recording past traffic events and analyzing them to generate reports. The software also allows real-time adjustments to the train schedule in the event of delays or accelerations during dispatch. It determines the capacity of each route based on demand, leading to improved decision-making. the software provides timetables and graphs for a comprehensive analysis of the metro network for each planner and decision-maker.

3.1. The basis for the design and implementation of a comprehensive timetable system for the Tehran metro

To create a comprehensive timetable system for the Tehran metro, essential information must be collected and implemented:

- Real-time information about the status of trains in the metro network.
- Ensure that the timetable system interacts with the PIS and PID systems in an accurate and timely manner in an integrated way.
- Determine the exact position of trains on the metro lines, by considering the distances and safety margins between trains.
- Ensuring the accurate and integrated connection of the comprehensive timetable system with the signaling systems, control and other trackside equipment in the metro network.
- Adjust the current train speed using ATS (automatic train supervision) mechanisms and ATC (automatic train control) systems in traffic control centers, ATP, or ATO (automatic train operation).
- Obtain real-time online feedback on the location and status of the metro train fleet, including passenger traffic at each station.
- Set up a secondary timetable to address traffic disturbances in the metro timetable system.

3.2. Technical details of the train fleet and metro lines in a Tehran metro comprehensive system

Designing a comprehensive timetable system requires careful consideration of technical factors:

- Determining the number of active trains on the line, taking into account whether they are motorized trains or trailers.
- It is important to allocate enough time to alight passengers or board them up at each station.
- Estimate the time required for emergency repairs to lines and trains by maintenance experts if technical defects occur.
- Ensuring that trains depart from stations on time and arrive at their destinations along the route on time.
- Facilitating the use of connecting trains at intersection stations by passengers.
- Implementing a flexible program alongside the fixed program in the system can increase efficiency.



• The technical parameters of Tehran's comprehensive metro scheduling system, as shown in Figure 3, are as follows:



Figure 3. Schematic layout of stations, railway blocks, and terminals in a case study (source: Tehran metro comprehensive system)

The kind of technical parameter	The amount in the Tehran metro
Service hours of the Tehran metro	5-23
The average distance between the stations in the study sample	600 meters
The average length of lines in the case study	400 meters
The average upward slope of the line in the case study	4 meters for every 1000 meters.
Maximum slope of the line in the case study	5%
Path length in a case study	39 kilometres
Number of stations in the case study	6
Platform length	140 meters
The power of train electric motors	132 and 180 KW
Bogie weight	747 Kg
The maximum prevailing wind speed in the case study route	40 m/s
Compressed air pressure in train	1013 KPa
The maximum temperature of the track	40°C inside the tunnel and 45°C outside the tunnel
Average train speed in the case study	55 km/h
The average width of the lane in the track	1435 mm
The energy received by trains (VCD)	VDC 750 or KV25 (overhead)
The average headway in the case study	6 minutes
Average train stopping time at the stations	45 seconds
The collection and method of energy reception	Third Rail
A collection of signs, signaling, and road circuits	Auto Block / ATC / ATO / ATS / CTC
Minimum Headway	2 minutes
Train starting side	R / L

Table 1. The technical parameters included in Tehran's comprehensive metro scheduling system

3.3. Brief of the Tehran metro comprehensive system problems

Designing timetables for a comprehensive metro system using conventional methods has several disadvantages. The central issue relates to human factors, which increase the risk of errors that can lead to financial losses and fatalities. When there is a rush or delay on individual trains, the station's local control centers make the necessary arrangements, which may not be relayed to central traffic control. Local control centers may lack a comprehensive overview of network traffic, which can lead to disturbances in other areas of the network. When optimizing train schedules, the primary goal is to minimize the time trains spend at a standstill while ensuring the safety of the network. Despite this, the current traffic management strategy often overlooks network safety due to the lack of a comprehensive approach to train management. Furthermore, the existing system is slow to respond to deviations. It is a result of the limited coverage of each local control center within a specific geographic area, along with the lack of a coordinated strategy to connect these centers with the main control center. It is important to recognize that stopping trains in a tunnel can have a more negative psychological impact on passengers than slower moving trains. If there is a need to expedite the process when a precedent is set, this should be accomplished by adhering to a safe and efficient timetable that keeps the train moving rather than stopping it in a station. This technique has not received enough attention in the current system, yet it could significantly enhance passengers' journey experience. Although trains are an effective communication system in transportation, it is currently not possible to use real-time information to make decisions about a train's arrival and departure times at the station as well as its current speed. The current Tehran Metro timetable system lacks efficiency. The comprehensive timetable has several issues that hinder the rail lines from reaching their optimum potential. One major problem with this system is its inability to respond to the following issues:

- passenger congestion or technical problems with the signaling system,
- emergency repairs on the line or not enough trains to cover the unexpected demand,
- equipment faults, e.g. in connection with the train computer,
- weather conditions, e.g. precipitation,
- rail accidents related to other vehicles or passengers,
- traffic issues leading to the occupation of lanes,
- shortage of operating personnel.

4. MATERIALS AND METHODS

This study, gathered data was collected from the Tehran Metro timetable system and capacity calculation methods from earlier research to design descriptive tables for railroad segments, stations and train fleets. Equations and functions were performed to calculate the capacity of the metro network by implementing the C++ programming language in an IDE environment. The study implemented a communication system between the software application and Tehran Metro database to estimate the capacity based on demand and allocate the fleet and equipment. The research's conceptual model is illustrated in Figure 4.





4.1. Calculation of the capacity of the comprehensive Tehran metro system

The capacity of the lines in the Tehran Metro lines is calculated using the UIC 405 equations. These equations enable the calculation of the network's actual capacity considering two lanes and traffic conditions (Liu et al., 2021). The UIC 405 relation is given as follows (Pan et al, 2020):

$$C = \frac{Time \ windows}{t_{fm} + t_r + t_{ZU}} \tag{1}$$

In the equation above, C illustrates the capacity of the metro network. The time windows account for the total available time minus the time blocked by lines. It is crucial to ensure tha t_r does not exceed the saturation factor, while t_{fm} represents the travel time of the train between the blocks. The calculation of t_{fm} can be established using the relationship outlined in the following section (Corman et al, 2015):

$$t_{fm} = \frac{\sum(n_{ij} + t_{fit})}{\sum n_{ij}}$$
(2)

The above equation t_{fit} represents the time interval between two consecutive trains in the metro network. The value n refers to the number of trains travel between stations i and j. To calculate the capacity of each rail section in the metro network, we first need to determine the travel time of each train on each line using computer simulators. Additionally, we need to update all train schedules and movement plans, particularly those that contain discrepancies, in the relevant equations. While the Tehran Metro's software for dispatching and receiving trains operates reliably under normal conditions, it has significant weaknesses when dealing with disturbances and interruptions. This is the primary issue addressed in this study. Therefore, we aim to resolve this challenge by implementing a rigorous approach in the comprehensive software for Tehran Metro timetables. This will enable us to accurately calculate passenger capacity during trips.



4.2. Mathematical modeling of the numerical equations of the comprehensive system of Tehran Metro timetables

To improve the Tehran Metro's timetable system, it is necessary to revise the mathematical equations. The first step is to determine the limits of the system. Timetables, together with the proper allocation of time buffers, can significantly impact the efficiency of the overall system. It is important to consider the actual needs and conditions of subway lines, including passenger waiting times, safety and security. The limitations of the problem include time frame and capacity. The time interval between two consecutive trains in the metro network is called the time interval of trains. If there are n trains on the main line, the time interval is equal to n_1 . The maximum interval time is determined by the passenger demand in the Metro, while the minimum interval time is determined by the safety factor (Kuppusamy et al, 2020). To calculate the value of the interval time in the metro network, the following equation is used:

$$Headway^{Maximum} \ge Headway_i \ge Headway^{Minimum}$$
(3)

In the above relationship, $Headway_i$ refers to the time interval between trains i and i+1. $Headway^{Minimum}$ Shows the lower limit of the time interval of trains and $Headway^{Maximum}$ shows the upper limit in determining the interval time. The stopping time of the train at the station, on the other hand, is determined by subtracting the arrival time from the departure time. In general, trains in the metro network run on an elliptical route between the origins and destinations in the lines adjacent to the upper and lower platforms. In this case, the duration of the train stop at each station is determined by the passenger demand of that particular station (Zhou et al, 2020).

In the defined context, *Headway_i* represents the time between trains i and i+1. The lower and upper limits of this time interval are denoted by *Headway^{Minimum}* and *Headway^{Maximum}* respectively. The duration of a train's stop at a station can be calculated by subtraction of its arrival time from its departure time. Typically, trains on metro networks follow an elliptical path between their origin and destination, with the upper and lower platforms adjacent to the lines. The train stop time at each station is determined by passenger demand at that station (Dolinayova et al, 2020). The stoppage time of a train at each metro station can be calculated using the following formula:

$$dwell^{Maximum} \ge dwell_i \ge dwell^{minimum}$$
(4)

The relationship mentioned above, $dwell_j$ refers to the time taken by the train to stop at station j. $dwell^{Maximum}$ is the maximum time limit for the train to stop at the station, while $dwell^{minimum}$ is the minimum time limit for the train to stop at the station. These limits apply to each train that stops at a specific station.

The total travel time for a train on a railway route is determined by subtracting the arrival time of the last train at the destination from the departure time of the first train at the origin. The lower and upper bounds of this travel period are affected by factors such as passenger demand and the method of operation for subway trains. This information has been documented in the provided reference (Wangai et al, 2020).

$$Total time^{Maximum} \ge Total time \ge Total time^{minimum}$$
(5)

In the above equation, "total time" refers to the entire duration of the train on the metro route. Additionally, "*Total time^{Maximum}*" and "*Total time^{minimum}*" represent the upper and lower limits of the total running time, respectively. The relationship between timetables and capacity in the metro network is highly intricate and intertwined, as stated in (Kavoosi et al, 2020). In the metro network, "capacity" refers to the maximum number of passengers that can board the train, based on the available space and permissible axial load. Each train has a maximum capacity limit, which is determined by the number of passengers per square meter of the train carriage, as mentioned in (Bauschert et al, 2017).

$$capacity^{Maximum} \geq capacity_i \geq capacity^{minimum}$$

(7)

The equation above refers to the capacity of train i $ascapacity_i$, and the upper and lower limits of a train are defined as $capacity^{Maximum}$ and $capacity^{minimum}$, respectively. Based on the information given, the optimization model for this problem can be expressed as follows:

Minimize C = $\sum_i c_i$

s.t

 $Headway^{Maximum} \ge Headway_i \ge Headway^{Minimum}$

i=1, 2, 3... n-1



$dwell^{Maximum} \ge dwell_k \ge dwell^{minimum}$	k=1, 2, 3 m-1
$Total\ time^{Maximum} \geq Total\ time \geq Total\ time^{minimum}$	(8)
$capacity^{Maximum} \ge capacity_j \ge 0$	j=1, 2, 3… n

In standard optimization problems, the decision variables usually include determining the duration of train stops in stations and on tracks, and managing the number of passengers on station platforms and in active trains. This study aims to improve timetables by effectively allocating capacity to meet passenger demand. Table 2 provides information on the symbols, indices and parameters of the problem.

Description	the symbol
Scheduled, documented, and original time according to the event e	M _{event}
The station is where the departure event occurs	Sevent
Disrupted train service for passenger transportation from origin to destination	v _{event}
The rail route for the train related to the event	υ _{line,e}
It is the type of train allocated to each event	$v_{type,e}$
The direction of the train in the movement operation	Direction
The event of consecutive train arrivals and departures	μ_{event}
The subsequent arrival and departure events of trains	E _{da}
Time description for scheduled events	Me
The station where the unplanned event occurred	S _{event}
The total number of unscheduled events	Edum
The routine of transferring passengers between railway lines at the metro intersection stations	Aboarding
Event of the train arriving at the station	E _{arr}
Event of the train arriving at the station	A _{waiting}
Availability of the event for trip groups	$u_{g,e}$
Number of activities related to A _{g.e}	N _{g,e}
Passenger groups waiting in the event of an unexpected occurrence	W _{before,g,e}
Time of the previous event	Xe
Passenger groups after the event	Wafter
Total number of waiting passengers	$\theta_{trains,g,e}$
The limited access at the intersection lines of the metro prevents some passengers from taking their planned route.	$ heta_{want,g,e}$
The number of passengers on the train	$\theta_{can,e}$
The remaining capacity of a train	$\theta_{remain,e}$

Table 2. Symbolic information regarding the dimensions and parameters of the problem

4.3. Management of train disruptions based on train capacity, wait times, and passenger volume

The redesign of train timetables can affect passenger waiting times and the overall flow of passengers. a model based on an "activity-event" network is used to address these factors. This model takes into account passenger waiting times on the platform, train capacity and routing measures for passenger movements at transfer stations.

4.4. "Activity-event" network in the passenger-oriented approach

An "event-activity" network has been created to enhance passenger flow within the metro system. This approach includes the analysis of waiting times and passenger load as well as the optimization of boarding and alighting events at the platforms. By managing these events effectively, we can improve passenger flow, resulting in a more efficient and enjoyable commuting experience.

4.5. Model assumptions

- Passenger demand is an important factor that needs to be considered while planning the operation of a metro network. This demand is measured at the origin, at the stations along the route and at the destination. In the Tehran metro network, the boarding rate is assumed to be time-independent, meaning that it does not change significantly within a relatively short period of time.
- Passengers usually choose the first feasible route after getting off the train, depending on the facilities available at the station, such as elevators, escalators, wheelchair access, etc.
- Sometimes some lines may not be accessible due to disruptions. In such cases, passengers wait at the station until the disruption has been rectified.
- The passenger flow in metro stations is assumed to be a continuous numerical value.

4.6. Events explanation

The network is designed to accommodate the continuous departure of trains, allowing for the modeling of passenger flow within a comprehensive framework. This allows for waiting time activities and passenger transfers to be factored in. Unlike the usual departure events in crowded subway systems, frequent exit events have an additional feature. In particular, train and platform events can be linked to exit events from the station to other lines and trains in interchange stations. This is called a μ_{event} . Here are some key features of the exit event:

- **M**_{event:} This is the planned and main time, considered according to the event *e* with or without interruption, depending on the event *e*.
- **S**_{event} station indicates the station at which the train is connected to the device's network.
- A train is given a v_{event} when it travels from the starting point to the destination.
- The train route for a particular train event, denoted byu_(line,e), specifies the observer's location, the start and end points of the train and all stops of the train along the route. The frequency of train dispatches is also considered when planning the train's operation and the train movement.
- The symbol $v_{type,e}$ is used to represent the type of train assigned to each event. There are two types of trains in the Tehran metro system: DC and AC. These are categorized based on the type of engine traction and operating speed.
- Track_{ke} refers to a specific section of the metro line assigned to an event. In the Tehran metro system, there are two parallel lines running north-south and east-west.
- Trains on line four of the Tehran Metro can run either eastbound or westbound during operation.
- The process of determining the dispatching times includes the arrival and departure events of trains from the departure stations related µ_{event}.
- The sequence of consecutive arrival and departure events of trains from metro stations is represented by E_{da} . An unscheduled event is represented for each station by the initialization of the waiting time variable and the passenger volume at that specific station.

The first occurrence of a train departure at a station based on time slots in an unscheduled event has the following specific characteristics:

- M_e: planning time is described for the coincidence of the scheduled event is the event on the basis of which the optimization time starts from this point.
- **s**_{event} refers to the station at which an unscheduled event took place.
- The total number of scheduled events is specified with Edum.

4.7. Activities

The activities between two continuous and distinct events are called a sequential event line, which shows the behavior of the participant at the time of the disturbance. The resulting relationships are shown as follows:

$$A_{boarding} = \{(e, e') : e \in E_{da}, e' \in E_{dep}, \mu_e = e'\}$$

(9)

 E_{dep} represents the event of the train leaving the station. The activities related to alighting from the train at intersection stations allow passengers to exit the noisy train and board another train to continue their trip.

$$A_{transfer} = \{(e, e'): e \in E_{arr}, e' \in E_{dep}, M_e - M_{e'} \ge L_a, \nu_e \neq \nu_{e'}, s_e = s_{e'} = e'\}$$
(10)

In equation (10), E_{arr} represents the event of the train entering the station, and passenger transfer activities also allow passengers to transfer from one train to another at intersection stations.

$$A_{waiting} = \{(e, e'): e \in E_{dum}, e' \in E_{dum}, M_{e'} - M_{e'} \leq \}$$
(11)

The activities related to waiting times and passenger congestion allow passengers to wait for the train at intersection stations with a single origin on a specific line.

4.8. Passenger-orientated metro network limitations

The domain of an "activity-event" network is described using some task functions such as boarding and alighting from a train, and waiting time at a station platform. In a study conducted by Khosrosarsheki and Moaveni (2021) on the elements, new relationships are established to calculate the waiting time and subsequent events for transferring to other trains in the transition lines.

4.9. Passenger routing at intersection stations

To effectively manage the rescheduling of trains, it is essential to provide an accurate description of passenger routes. These routes can be determined and adjusted at intersection stations using the exact coordinates of the origin and destination stations. In this study, we assume that the demand for trips between the origin and destination stations does not depend on time. Consequently, if there are no significant changes in passenger volume, any disturbances can be addressed within a complete time frame. In this study, passengers with similar origins and destinations for a specific period are grouped together, which may sometimes have the following characteristics:

- O_g is the origin of passenger group g and D_g is the destination of passenger group g.
- The set of passenger groups on a particular subway line is denoted by g.
- A route between start and destination is defined for each passenger group.
- These routes are mainly defined along the rail lines and between the events of leaving the stations along the route and arriving at the destination station.
- The route of each passenger group is mainly determined based on time series and historical data; therefore, the activities of each passenger group at each rail event are denoted by A_{g,e}

Some routes may become unavailable due to rail disruptions and rescheduling measures. Therefore, some studies, such as Schanzenbacher and colleagues (2021), have assigned a binary variable to a route of a particular passenger group g, which is used for successive events e. Thus, if the route is available for passenger group g after event eug,e, it has the value one, otherwise it has the value zero. Passenger routes are only available when all route activities are operational and trains continue to run normally according to the scheduled timetable.

$$u_{g,e} \le (1 - c_e), \forall (e, e') = a \quad \in A_{g,e} \forall g \in G, \forall e \in E_{da}$$

$$\tag{12}$$

$$u_{g,e} \ge \sum c_e - (N_{g,e} - 1), \ \forall (e,e') = a \in A_{g,e} \ \forall g \in G, \forall e \in E_{da}$$

$$\tag{13}$$

In the above relationship, $N_{g,e}$ refers to the number of activities related to $A_{g,e}$. These robust routines guarantee that for a passenger route at intersection stations, one of the routes is canceled ($c_e = 1$) in which case ($u_{g,e} = 0$). Limits13 guarantees that the passenger route is available, and then $u_{g,e} = 1$, and this is for the duration in which none of the transfer activities were canceled. It should be mentioned that in the current research, which determines the amount of variability $u_{g,e}$, it is the activities related to $A_{g,e}$. This process facilitates the transfer of a designated group of passengers during the entire boarding process, including boarding and disembarking from the train while waiting on the platform. It also includes transfers to other intersecting lines in the station.

4.10. Passenger waiting time

When a departure event occurs, the ratio of passengers on the platform changes compared to the passengers boarding the train or departing to other lines. It is assumed that the rate of passengers boarding the platform is constant. In this scenario, the number of passengers waiting on the platform can be calculated for each group before the train departs.

In the above relationship, x_e is the time of the previous event, η_g is the number of passengers entering the station at a certain moment, and w_{after} for a specific passenger group. $\theta_{trains,g,e}$ represent the total number of waiting passengers for each trip group.

$$w_{before,e} = \sum_{g \in G} w_{before,g,e} \quad , \ \forall e \in E_{da}$$
(15)

If an unplanned and unexpected event occurs during the process of train movement, some passengers on the train may reach their destination by the same means of transport. However, the number of remaining passengers who want to reach their destination by train under unpredictable conditions is represented by a continuous variable such as $\theta_{want,g,e}$. This arithmetic value is the number of passengers who do not use the desired route in the crossing lines due to restricted access. The total number of passengers on the train during the time the train leaves the station can be determined by the ratio between the total waiting passengers for all passenger groups:

$$\theta_{want,g,e} = \sum_{g'} \quad \forall \ g \in G \in E_{da} \tag{16}$$

If a train is at the threshold of its maximum capacity and none of the passengers can board the train, then the calculation of the number of passengers on the train can be expressed with the variable $\theta_{can,e}$. In this case, the number of passengers who can board the train is equal to the minimum volume of passengers waiting on the platform and the volume remaining on the platform:

$$\theta_{can,e} = \min(\theta_{can,e}, \theta_{remain,e}), \forall e \in E_{da}$$
(17)

In equation (17), $\theta_{remain,e}$ is the observer of the remaining capacity of the train in event *e*. When a delay event occurs, several passengers board the train. Subsequently, the number of passengers remaining on the passenger platform after leaving the train is calculated from the following equation:

$$W_{after,g,e} = W_{befor, g,e} - \theta_{can,e} \forall g \in G \in E_{da,e}$$
(18)

Therefore, the remaining capacity in a train depends on the total capacity, the number of passengers that get off, and the total number of passengers inside the train coach:

$$\theta_{remain,e} = C - \theta_{after,g,e'} + \theta_{alight,e}, \quad \forall g \in G \in E_{dde}$$
(19)

In the above relationship, *C* is the maximum of the total capacity of the train, and $\theta_{after,g,e'}$ indicates a group of passengers whose numbers are left behind after the train leaves the station. to estimate the number of remaining passengers after some alight from the train, we can assess the target groups of passengers.

5. CASE STUDY

The case study goes back to the southern half of line one of the Tehran Metro. the southern half of line one of the Tehran Metro, including the high-traffic rail sections from Aliabad metro station — Jovanmard Kasab; Jawanmard Kasab – Shahre-rey; Shahre-rey — Refinery (Fathabad); Refinery (Fathabad)—shahed; Shahed – harame motahar. according to the outputs of the comprehensive system designed for managing and controlling the metro schedule, the underground sections of the metro are divided into three main categories. In the first section, there are several blocks where both the number and volume of passengers are high. However, only a limited number of trains run between these blocks in relation to passenger demand. This information is shown in Figure 5.



Figure 5: schematic view of the southern section of line 1 of the Tehran metro, which is based on the output of the comprehensive system of the timetable

Suppose that one of the trains in the north-south direction of line one of the Tehran Metro has a problem in the western part of the platform of the Aliabad Metro. In this case, if the deviation from the original table is 700 seconds, this system considers twice the specified value (1400 seconds) as the time to restore the normal state. Consider the duration of the train breakdown and its potential to cause delays at other stations along line 1, especially at intersecting stations such as Shahed station. In terms of implementation, the action included in the system will not have the required effectiveness. Thus, to deal with this problem in the mentioned system, two alternative tables with different time strategies can be set. Figure 6 shows the initial part of the timetable for the standard operation of the lines, which is based on the established plans for the Tehran Metro. The scheduled timeline for the second part covers contingencies.



Figure 6.The line graph of train movements in the scheduled time horizon in the south half stations of Line 1 (source: outputs of the comprehensive Tehran metro timetable system)

A comprehensive system for setting metro timetables is an expert system that sits in place of a human agent. At any moment, it can not only control the instantaneous movement of trains under normal traffic conditions, but also suggest a speed for trains with delays. This, in addition to maximising compliance with the passenger train schedule, can compensate for the disruption caused by noise to the metro network fleet (De Leenheer et al, 2016). Figure number 7 shows the southern section of line 1 of the Tehran metro.



Figure 7. The southern part of Line 1 of the Tehran Metro

Table 3 shows the current timetables of the southern half of Tehran metro Line 1 based on the comprehensive timetable system:

Station name	Aliabad	Javanmard	Share-Rey	Fatehabad	Shahid	Harme motahar
Arrival time (second)	-	150	243	405	562	669
Departure time (seconds)	0	185	273	438	556	669
Stopping time at the station (seconds)	-	35	30	32	30	30

Table 3. Overview of the Current Timetable for the Southern Section of Line 1 of the Tehran metro

Table 4 shows the dynamic parameters related to the management of metro trains based on the comprehensive system of the timetable in the southern half of line one.

Rail track name	Distance (in meters)	Train travel time (seconds)
Aliabad - Javanmard	1580	153
Javanmard – Shahre-rey	920	95
Shahre-rey - Fathabad	2100	190
Fathabad - Harme motahar	840	160

 Table 4. Dynamic parameters related to the management of the subway trains based on the comprehensive system of the timetable in the southern half of Line 1

In Table 5, the uncertainty rate for each of the stations in the southern half of Tehran Metro Line 1 is shown based on the comprehensive timetable system.

Station name	Lower limit (passenger/second)	Upper limit (passenger/second)
Aliabad	0/6	0/8
Javanmard	0/4	0/7
Share-Rey	0/8	1/2
Fathabad	0/2	0/3
Harme Motahar	0/5	0/7

Table 5. Uncertainty rate for each station in the southern half of Tehran metro Line 1 based on the comprehensive timetable system



Table 6 shows the capacity analysis of the southern section of Tehran metro Line 1 about passenger congestion in the comprehensive timetable system.

Rail track name	The amount of available capacity (percent)
Aliabad - Javanmard	100
Javanmard – Shahre-rey	100
Shahre-rey - Fathabad	85
Fathabad - Harme motahar	65

Table 6.The capacity of the southern half of line one of the Tehran metro, based on the outputs of the comprehensive system of the timetable

In the metro network, schedule optimization aims to minimize the waiting time of passengers on platforms, reduce the negative impact of delays and improve factors such as the energy consumption of the rail system. Research shows that certain objectives often conflict with each other. However, in metro transport, adjusting timetables can greatly affect both passenger waiting times and delay management. Therefore, it is essential to establish timetables that create a harmonious balance between these different objectives. The sensitivity analysis shows that due to the variable parameter of passenger demand, the waiting time of passengers at the platform has a greater weight than usual (Frutos Bernal and Martín, 2022). As a result, in the proposed approach to optimize the comprehensive system of Tehran metro, it is recommended to adjust the tables be adjusted so that the passenger demand is first calculated. This is done by providing a set of solutions and eliminating the inferior options by comparing the solutions. Providing a better strategy could improve the current schedule in the comprehensive system of Tehran metro timetable. At the opposite end of the spectrum, train frequency and passenger demand are influenced by time. Field research conducted along the southern half of Tehran Metro Line 1 indicates that redesigning the comprehensive schedule for Tehran Metro can reduce passenger waiting time by 26.58 minutes. This adjustment would lead to more effective passenger transfers and help prevent conflicts within the metro network. Table 7 shows the improved time gap after applying the robust optimization approach.

Fathabad - Harme	Shahre-rey -	Javanmard – Shahre-	Aliabad -	Rail track
Motahar	Fathabad	Rey	Javanmard	
142	152	130	78	Train travel time

Table 7. Time interval of trains after optimization (seconds)

Table 8 shows the improved timetable after applying the robust optimization approach in the comprehensive timetable system:

Station name	Aliabad	Javanmard	Share-Rey	Fatehabad	Shahid	Harme
						motahar
arrival time (seconds)	-	141	233	394	518	658
departure time (seconds)	0	189	285	463	563	708
Stopping time at the station (seconds)	-	37	33	35	33	36

Table 8. Data of the proposed timetable of the southern section of Tehran Metro Line 1 after optimization (seconds)



Figure 8. Findings obtained from computational operations to optimize the timetable achieved in the comprehensive Tehran metro system.

In this study, the data recorded by the system components were used to assess the validity of the processes and the correctness of the connections between the model's inputs and outputs. It is possible to refer to the total travel time or the duration that each rail segment is blocked due to issues such as signal system failures or passenger congestion. In this model, the system inputs the distribution functions of travel time and train schedules in each rail section and station. The travel time for each train in the rail sections is determined based on the adherence to the timetables. In the validation test, we compare the performance of the operating system with the real running times of the trains in the metro network By analyzing the average travel times of the metro trains based on the simulation outputs. A test setup confirmed the validity of the model, using the evalfis function to evaluate the proposed system in the MATLAB software. Assuming a train is located 800 meters away from a station in the southern part of Tehran metro Line One, it has a delay of 7 minutes. The aim of this process is to evaluate the response of the system and determine its recommendations for train speed. First, we create an object called "delay" from the noise train system based on the relationships previously defined on a part of the computer disk. Then we execute the following code:

(20)

>>evalfis ([7],delay)

ans = 69.0121

6. CONCLUSION

By integrating the comprehensive timetabling system with the metro network's traffic information database, we were able to calculate capacity based on variable demand and provide solutions for disruptions. Tehran metro operators must adapt to meet passengers' changing demands by managing dispatches effectively and considering train sizes. To minimize the waste of resources on the metro network, we should employ the fewest number of employees and train crews at the stations. It is important to maximize the efficiency of the Metro network through proper management of traffic disruptions. The strategy outlined in this research aims to enhance the Tehran subway timetable system. It can reduce the calculation time by about 15-25%. Another important aspect of the proposed approach is the ability to effectively manage conflicts between arriving and departing passengers, which can significantly improve the overall efficiency of the metro network. This is done in order to minimize depreciation and amortization costs without compromising the performance of other parts of the metro network in terms of fixed and mobile facilities and equipment. It is important to explain that several factors affect the routing and movement of trains. These include the technical characteristics of passenger trains and metro lines, which can enhance the reliability of schedules against potential network disturbances. In this research model, corrective actions to address disruptions at stations along the route, including origin stations and incoming dispatches from terminals, were assessed and investigated. Based on this, a new method for calculating network connectivity using the event-activity network approach was presented. The negative impact of delays on passenger waiting times were not considered. Furthermore, even if the number of passengers is calculated, a train cannot accommodate all passengers unless it has excess capacity.

Consequently, changes in passenger demand due to rescheduling affect how we understand station congestion and a detailed overview of passenger flow can be provided.

CONFLICT OF INTEREST

The contact author has declared that none of the authors has any competing interests.

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