Evaluating Capsize Boundary of Damaged Ro-Ro Ship Using Chaos Theory and Probabilistic Analysis

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Instability in intact condition occurs when a ship's motion changes periodically with time i.e. frequency of motion will match with free- roll frequency or multiples. In case of damage stability, the instability occurs due to flood water sloshing, a series of vibration occur along with free surface effect, which changes the metacentric height of ship. Mathieu's effect is the linear theory behind instability. Probabilistic damage stability analysis is performed in damage cases for the Ro-Ro vessel to understand the most critical loading condition. Phase plots and radial plots are used to find critical stability parameters for ship at intact condition. A new method for modeling the nonlinear dynamic behavior of ship in the chaotic flooded condition is developed and the results indicate that the vessel behaviour of flooded ship is similar to the Duffings oscillation equation and the stability conditions can be captured by this model.

KEY WORDS

~ Stability
~ Capsize
~ Dynamics
~ Vibrations
~ Chaos
~ Damage
~ Probability

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1. INTRODUCTION

Stability is a major concern for massive ships like Ro-Ro vessels, which are known to have large variation in its metacentric height (GM) during navigation. Due to ship motion, an added acceleration is developed which creates a series of bifurcations in velocity potential, such that the flow around ship and its motion are disturbed. The metacentric height varies at this point, and it develops a perturbation moment where damping coefficients play a significant role (Yang et al. 2018). Since the roll response of vessel in beam sea condition is large, the roll damping effects are higher, and it controls the overall roll amplitude. In the present study, the frequency at which resonance occurs in linear condition is assessed, which is found to be influenced by the damping coefficients. The resonance occurs when the excitation frequency is multiples of roll natural frequency and hence, their ratio varies as ¼, ½, 1, 2, 4 etc. The most common method for predicting resonant rolling is Ince-Strutt diagram which shows stability and instability regions of ship motion (Butikov, 2018). In a dynamic system with periodically varying excitation forces, the equation of motion takes the form of Mathieu's equation . Mathieu's equation is an ordinary differential equation having real coefficients, whose solution depends on the stability parameters and not on the boundary conditions. The phase plot developed brings out the effects of variation of metacentric height and damping factor on stability under resonant condition. Table 1 gives the nomenclature considered in the present study. Under damaged condition, resonant behaviour is more intensive and will promote rapid flooding and sinking of vessel, giving less time to evacuate the ship. The damaged ship represents a dynamic system with chaotic behaviour.

Table 1. Nomenclature

Parameters	Symbol
Accumulated Flood water forces	U
Added mass coefficient	A_{33}
Added mass moment of inertia coefficient	A_{44}, A_{55}
Angular Acceleration	$\ddot{\phi}$
Angular Velocity	$\dot{\phi}$
Breadth Moulded of ship	B
Change in metacentric height	δGM
Damped frequency	ω_d
Damping factor	ξ
Damping force coefficients	B_{33}
Damping moment coefficients	B_{44}, B_{55}
Displacement	Δ
Draft	d
Length between perpendiculars	I_{pp}
Mean metacentric height	GM_m
Mean roll variation frequency	ω_m
Metacentric height	GM
Metacentric height of ship in waves	GM_a
Mass moment of Inertia	I_{xx}
Aft Perpendicular	AP
Prismatic coefficient	C_p
Righting Arm	GZ

Roll Angle	ϕ
Roll variation frequency	ω_a
Stability characteristics	α, μ, γ
Stability Parameters	ε, ϕ, ψ
Vertical Centre of gravity	VCG

The motion equation of damaged ship can be considered in the form of Duffing equation. The Duffing equation is a second order differential equation which represents damped oscillation driven by an external force. This model will be useful in predicting the stability and capsize behaviour of a damaged ship. The differential equation was coded in MATLAB, which gave the phase plots corresponding to the calculated stability parameters and the vibratory nature of the ship was captured.

2. REVIEW OF RELATED WORKS

The occurrence of rolling in ships is a threat to dynamic stability. Hence, the angle of heel is considered as a significant criteria for stability assessment as per International Code of intact stability (2008). Conditions of large stability variations are influenced by wavelength, wave height, metacentric height, righting arm, shape of hull, stern and bow flares. Parameters affecting ship stability have been studied by many researchers and a few of them are discussed in this section.

The angular acceleration and angular velocity have given an idea of the sea keeping behaviour of ship. France et al. (2002) investigated containership using experimental and numerical analysis to observe rolling in head seas during capsize. It provided an insight into the sea-keeping behaviour of a capsizing ship. Korkut et al. (2004) performed experiments to find six degrees of motion of intact and damaged Ro-Ro ship for a set of wave heights and frequencies in regular sea. The analysis gave responses in head, beam and quarter sea conditions and found that the motions were affected by wave directionality and frequency range. A guide was developed by Shin et al. (2004) to find the roll frequency of container ships using Large Amplitude Motions Program (LAMP) in irregular waves and suggested an anti-roll tank to reduce rolling. A third order mathematical equation was introduced by Neves et al. (2005) to describe extreme roll on ship due to periodic restoring moment. They found coupling coefficients of heave and pitch motions for any range of external excitation forces. It was concluded that coupled ship motions cause non-linear effects, especially when the metacentric height is low. Chang (2008) used strip theory on Ro-Ro vessel to determine response amplitude operators (RAO) in heave, pitch sway and yaw motions. The roll and surge motions were analysed with Grim's effective wave concept to determine righting moment in seaways. Francescutto (2015) used Mathieu's equation and found the roll to be affected by tuning ratio. Capsize roll instabilities were difficult to be captured experimentally, where wave generation at extreme environmental condition was the most uneconomical part. Mathieu's equation of motion was used to assess roll in head seas by Arndt et al. (2016). They found the motion response of ship model under linear and nonlinear conditions corresponding to varying propeller rotation speed, angle of rudder, etc. A maximum roll angle of 30° was investigated, and the stability parameters obtained were found to be within boundaries of instability of Mathieu's equation.

Damage stability experiments gained impetus during the 1970's, studying the effects of water entering through damaged region and other various methods aimed at improving the restoring capacity of ships. Methods to measure the size of damage opening and how it affected the damage stability indices, such as attained and required indices, were developed by Kambisseri et al. (2003). However, very few researchers have formulated stability theories based on motion equation of damaged ship, as it is a function of a number of parameters. Stability during roll motion of damaged ship containing flood water was discussed by Santos and Guedes Soares (2010), using time domain simulation. Vassalo et al. (2004, 2013) suggested a combination of parameters like

GM, GZ, angle of heel, etc. to study capsizing of ship at forward speed. Santos and Guedes Soares (2019) predicted progressive flooding condition using time domain analysis. The parameters considered include quantity of flood water, ship roll response, shearing and bending in ship beam girder, length of ship, location of damage, extend of flooding, etc. Flooding phenomenon is reported to be highly unpredictable and a non-linear condition. In the present study, the relationship between roll motion of intact and damaged Ro-Ro vessel is highlighted.

3. THEORETICAL BACKGROUND

Consider ship with motion responses as shown in Fig. 1. The equation of motion (Santos et al., 2010) describing the flooded ship under damaged condition is given in Eq. 1:

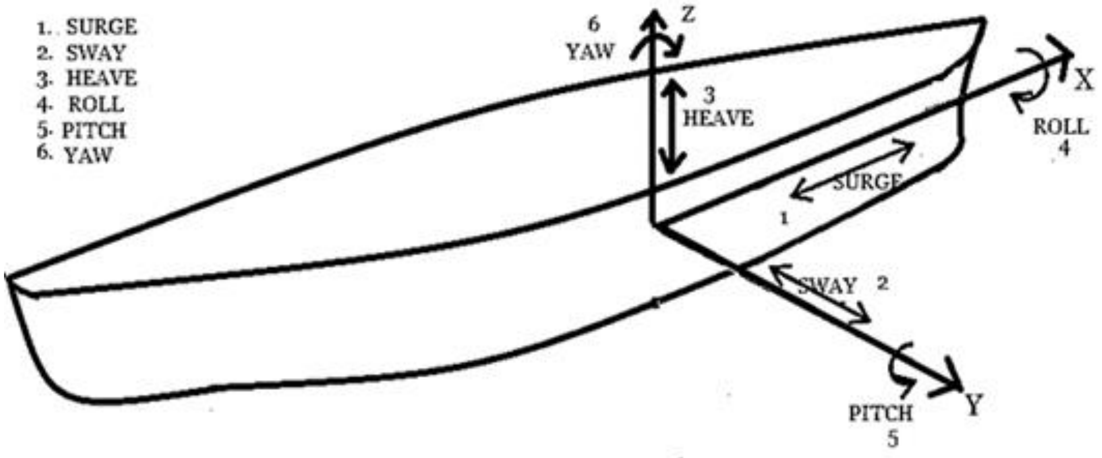


Figure 1. Co-ordinate Axis

$$\sum_{j=1}^{6}[\mathrm{M_{ij}} + \mathrm{A_{ij}} + \mathrm{A_{ij\text{-}AC}}]\ \ddot{x} + [\mathrm{B_{ij}} + \mathrm{B_{ij\text{-}AC}}]\ \dot{x} + [\mathrm{C_{ij}} + \mathrm{C_{ij\text{-}AC}}]\,x = F_i^E + F_i^{AC} \qquad (1)$$

Where, $\mathrm{M_{ij}}$ = Mass matrix; $\mathrm{A_{ij}}$ = Added mass of ship; $\mathrm{A_{ij\text{-}AC}}$ = added mass due to accumulated water; $\mathrm{B_{ij}}$ = damping coefficient of ship; $\mathrm{B_{ij\text{-}AC}}$ = damping due to free surface effect; $\mathrm{C_{ij}}$ = hydrostatic restoring force; $\mathrm{C_{ij\text{-}AC}}$ = restoring force due to free surface effect, F_i^E =Wave excitation force matrix; F_i^{AC} = Accumulated water force matrix.

The intact ship with metacentric height GM has a roll motion equation as given in Eq, 2:

$$\ddot{\phi} + 2\xi\dot{\phi} + (\omega_{\mathrm{m}}^{2} + \omega_{\mathrm{a}}^{2}\cos(\omega t)\,\phi = 0 \qquad (2)$$

Where ϕ is roll amplitude, ξ is damping factor. Added mass arises due to ship hydrodynamics and is influenced by vessel displacement, motion, density of fluid, and hull form.

$$\omega_m = \sqrt{\frac{\Delta GM_m}{I_X + A_{44}}} \quad \omega_a = \sqrt{\frac{\Delta GM_a}{I_X + A_{44}}} \qquad (3)$$

Where *Δ* is ship displacement, I_x is transverse mass moment of inertia, and A_{44} is added mass moment of inertia in roll, GM_m is mean metacentric height, GM_a is metacentric height with respect to waves.

When vessel is subjected to linear waves, the GM value varies and is represented as:

$$GM\,(t) = GM_m + GM_a\ \mathrm{Cos}\,(\omega t) \quad (4)$$

To transform into Mathieu's equation, we introduce a dimensionless parameter:

$$\frac{d^2\phi}{d\tau^2} + 2\xi\frac{d\phi}{d\tau} + (\omega_m + \omega_a\cos(\tau))\phi = 0 \quad (5)$$

$$\tau = \omega t\ ; \qquad \mu = \frac{2\xi}{\omega}\ ; \qquad \overline{\omega_m} = \frac{\omega_m}{\omega}\ ; \qquad \overline{\omega_a} = \frac{\omega_a}{\omega} \quad (6)$$

Final roll damped Mathieu's equation in terms of variation of metacentric height is given by:

$$\frac{d^2\phi}{d\tau^2} + \mu\frac{d\phi}{d\tau} + (\alpha + \gamma\cos(\tau))\phi = 0 \quad (7)$$

$$\mu = \frac{B\omega_d}{\omega(I + A\omega_d)}\ ; \qquad \gamma = \frac{\delta GM}{GM_0}\alpha\ ; \qquad \alpha = \left[\frac{\omega_m}{\omega}\right]^2 ; \qquad \alpha = \overline{\omega_m}^2\ ; \qquad \gamma = \overline{\omega_a}^2 \quad (8)$$

Normalized Added mass coefficients (A_{ij}') and damping coefficients (B_{ij}')are non-dimensionalised as:

i=1-3 and j=1-3 : $A_{ij}' = \frac{A_{ij}}{\rho\nabla}$ $B_{ij}' = \frac{A_{ij}}{\rho\nabla\omega}$, *i=1-3 and j=4- 6:* $A_{ij}' = \frac{A_{ij}}{\rho\nabla L}$ $B_{ij}' = \frac{A_{ij}}{\rho\nabla L\omega}$ (9)

i=4-6 and j=1-3 : $A_{ij}' = \frac{A_{ij}}{\rho\nabla L}$ $B_{ij}' = \frac{A_{ij}}{\rho\nabla L\omega}$, *i=4-6 and j=4-6:* $A_{ij}' = \frac{A_{ij}}{\rho\nabla L^2}$ $B_{ij}' = \frac{A_{ij}}{\rho\nabla L^2\omega}$ (10)

Often the capsizing of the ship under damaged condition is considered as a nonlinear dynamic system with periodically forced oscillations. Hence, the forced Duffiing's equation (Eq.12) and Vanderpol's oscillation (Eq.13) principles are considered. Here, $\alpha, \lambda, \gamma, \omega$ are constants and *U* is the external flood response actuator and $P\cos\omega t$ is wave excitation force.

$$\ddot{\phi} + \mu\dot{\phi} + [\alpha x + \gamma x^3]\phi = P\cos\omega t + U \quad (11)$$

$$\ddot{\phi} + [\lambda(x^2 - 1)]\dot{\phi} + \phi = P\cos\omega t + U \quad (12)$$

Thus the solution to capsizing ship under flooded condition and that which experiences the chaotic state is as represented as:

$$\begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{bmatrix} = \begin{bmatrix} x_2 \\ -\gamma x_1^3 - \mu x_2 - \alpha x_1 + P\cos\omega t + U \end{bmatrix} \tag{13}$$

$$\begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{bmatrix} = \begin{bmatrix} x_2 \\ -\lambda(x_1^2 - 1)x_2 - x_1 + P\cos\omega t + U \end{bmatrix} \tag{14}$$

This model is used in the present study to define the damaged ship motion and it is modelled in Section 7 to obtain the stability regions of the capsize ship model.

4. HYDRODYNAMIC CO-EFFICIENT OF RO-RO VESSEL

Principal particulars of Ro-Ro vessel considered is listed in Table 2. The motion response analysis and validation of Ro-Ro vessel (Fig 2) was performed and reported in Poonam and Shashikala [17].

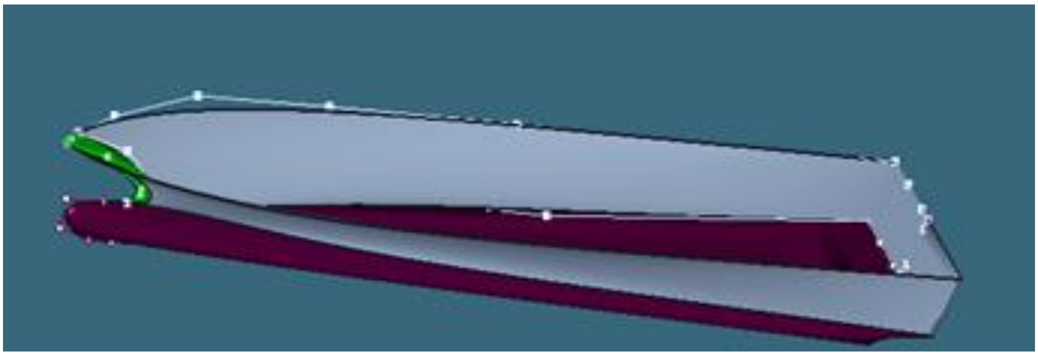


Figure 2. Hull form of Ro-Ro vessel

Table 2. Hull Form Properties of Ro-Ro vessel

Parameters	Unit	Dimension
Displacement	kN	164800
Volume (displaced)	m^3	16391
Draft at AP	m	6.5
Block coefficient	-	0.561
Prismatic coefficient	-	0.604
Length of ship	m	187
Length b/w perpendiculars	m	173
Breadth of ship	m	26
Mid section coefficient	-	0.929
Radius of gyration of Roll	m	12.25
Radius of gyration of Pitch	m	45.22
Depth to public spaces deck	m	15.7
Water plane coefficient	-	0.794
Height of metacenter above keel	m	14.08
Height of C.G above keel	m	11.04
Metacentric height	m	3.04
Longitudinal position of CG from AP	m	78.73

The calculated non dimensionalised added mass and damping coefficients of the vessel are as shown in Fig. 3(a-d).

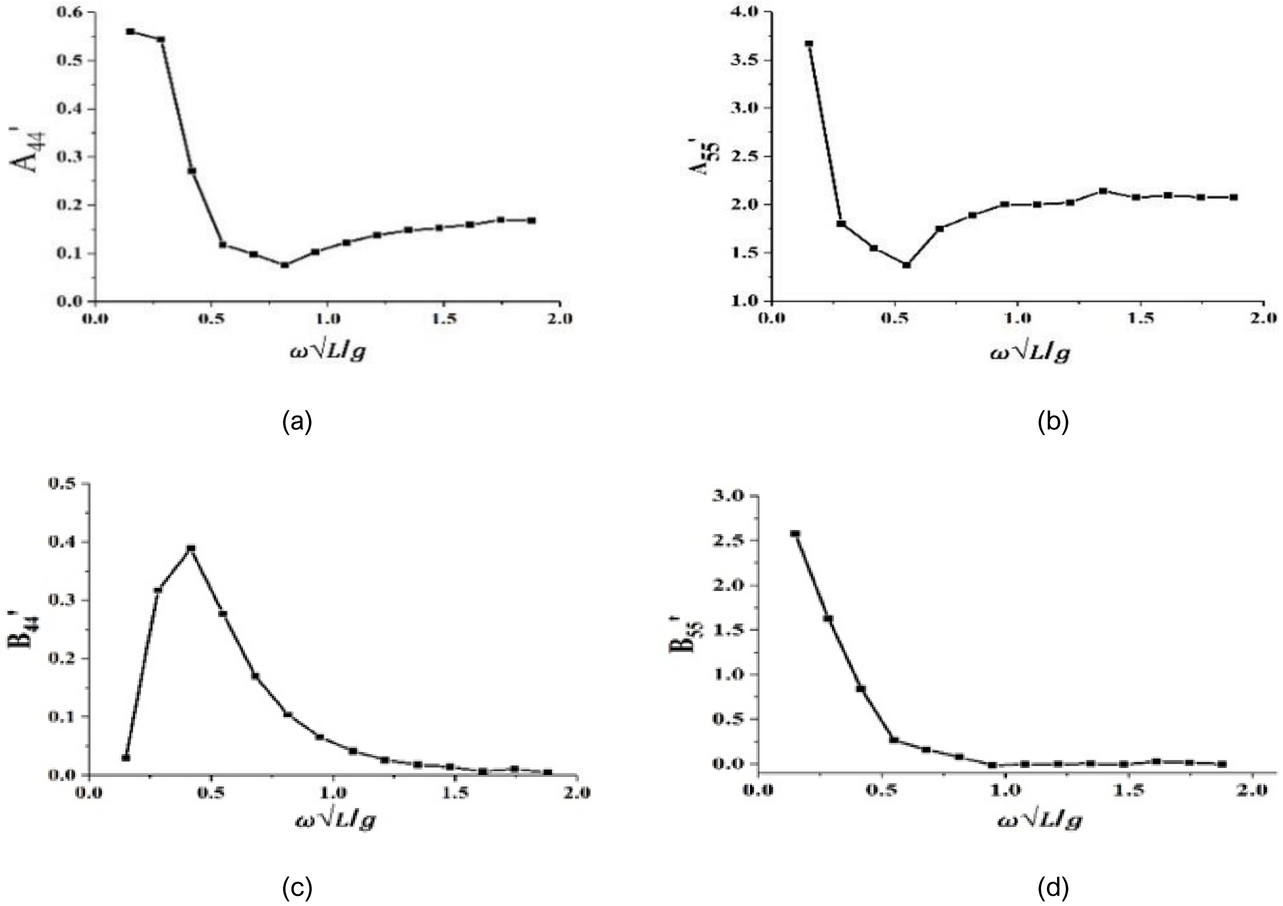


Figure 3. Dimensionless hydrodynamic coefficient of Ro-Ro at zero forward speed in quarter sea condition (a) Added mass in roll (b) Added mass in pitch (c) Damping in roll (d) Damping in pitch

As the ship is roll dominated, the ship experiences smaller damping force when compared to pitch motion. It has been found that, as the wave frequency increases, the added mass decreases. The effects of hydrodynamic coefficients on pitch and roll are found to be significant enough to cause stability variations. Hence the stability parameters are calculated in the following section to determine the most critical stability condition.

5. STABILITY VARIATION OF RO-RO VESSEL IN INTACT CONDITION

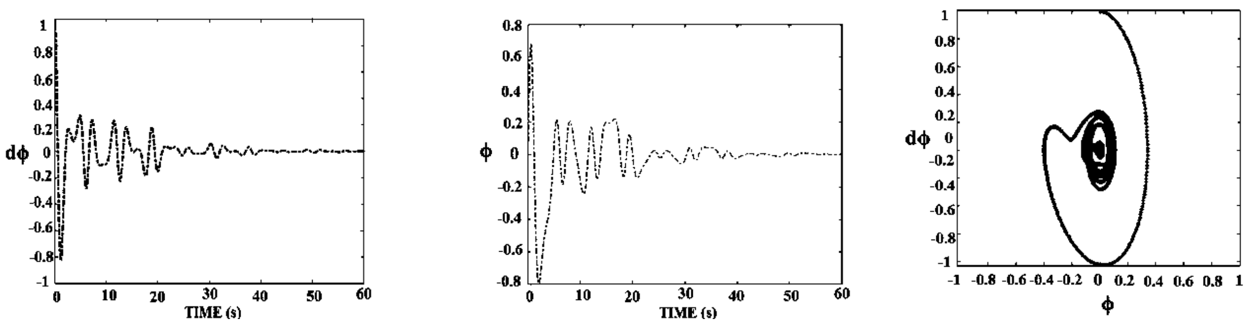
A change in phase plots shows evolution of dynamic behaviour of ship with respect to time, revealing the stability characteristics. Phase diagram changes for different values of stability parameters of a particular roll dynamic equation (Insperger, 2003). The phase plots obtained by numerical modelling of Mathieu's differential equation are discussed for different cases, with respect to stability parameters which define stability and instability ranges. The differential equation (Eq.7) is a damped Mathieu's equation whose eigenvalues will move away from the fixed point when it has a positive real part and the system becomes unstable. It moves back to steady state if it has a negative real part and the system becomes stable. It will oscillate around the steady state if it has an imaginary part. The system will remain in position with a constant amplitude if eigenvalue is zero. The stability conditions were applied randomly on Mathieu's differential equation coded in MATLAB to obtain the phase plots corresponding to roll angular velocity and roll angle. The α, γ, μ values were established by randomly applying a set of values of stability parameters, to obtain the values at which the motion equation falls on the stable and unstable region.

Roll response of vessel approaches stable region with damping when α =1, γ =0.05, μ =0.003. A vibratory, non-periodic response occurred when α =1.25, γ =0.45, μ =0.003 and an unstable, damped, decaying and sinusoidal response when α =1.3, γ =0.5, μ =0.003. To understand the stability behaviour of Ro-Ro vessel, phase plots are plotted and regions of instability are found out. Critical roll damping condition induces oscillation in vessel, variation of roll angle and metacentric height. Wave travels along the hull of vessel as crest and trough alters GM along the process. Metacentric height varies for each wave frequency, hence corresponding variation in GM was estimated for Ro-Ro vessel model. Non-linear restoring coefficients corresponding to heave, roll, and pitch of the ship in waves were calculated. Related stability factors were calculated from Eq 8 and 9 from hydrodynamic parameters estimated numerically from MAXSURF. It was applied on the MATLAB code, to observe stability variation. The α, γ, μ values calculated are listed in Table 3.

Table 3. Calculated values of α, γ, μ

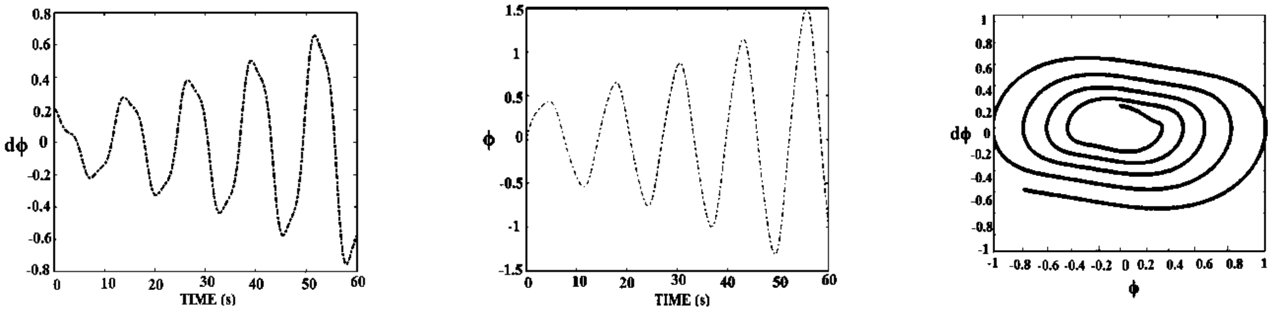
Case:	ω (rad)	α	γ	μ
1	0.2	3.783	4.239	0.175
2	0.39	1.204	0.077	0.143
3	0.78	0.204	0.182	0.120

Variation of roll velocity $\dot{\phi}$ with time is plotted in Figure 4-a, 5-a and 6-a, for all three cases listed in Table 3. Similarly, variation of roll angle ϕ with respect to time is plotted in Figure 4-b, 5-b and 6-b. Stability changes and the ship behaviour can be studied using the phase plots obtained in Figure 4-c, 5-c and 6-c . All the three cases show the effect of damping. Case 1 gives damping with vibration as shown in Fig 4. It shows a large roll angle initially, which converges with nonlinear vibrations to zero by gradual decay. In Case 2, values of α and γ continue oscillations which do not decay in the presence of damping as shown in Fig 5 (c). The parametric oscillation occurs and the whole system moves away from stability region. This occurs when excitation frequency is twice the natural frequency of vessel. Oscillation induces huge instability to the whole system and the vessel will collapse at this point.



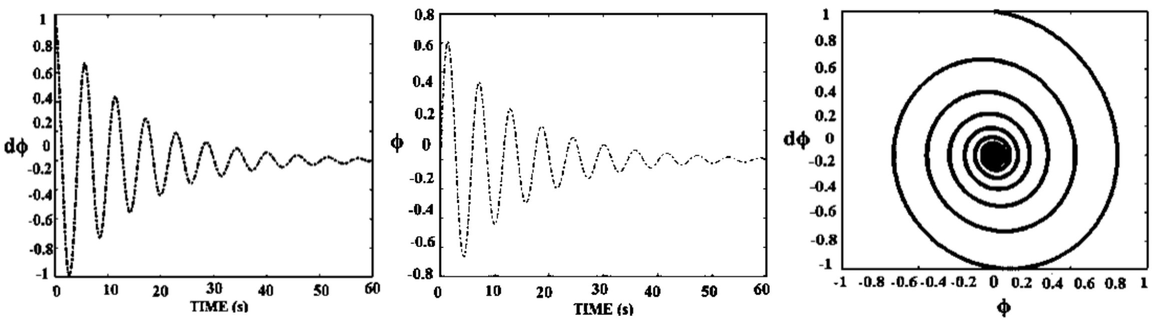
(a) (b) (c)

Figure 4. Damping with vibration-Case 1(a) angular velocity (b) roll (c) phase plot



(a) (b) (c)

Figure 5.Parametric oscillation - Case 2 (a) angular velocity (b) roll (c) phase plot



(a) (b) (c)

Figure 6. Damping without vibration- Case 3 (a) angular velocity (b) roll response (c) phase plot

Fig. 6 shows the Ro-Ro ship undergoing a few roll motions and then moving towards a stable state (Case 3).Thus the roll response of Ro-Ro ship under intact condition was obtained by including the GM variation on the stability parameter. To find the variation of GM under damaged condition and to obtain the most critical damage location based on load cases and damage cases, the probabilistic analysis was performed, which is discussed further.

6. PROBABILISTIC DAMAGE STABILITY ANALYSIS

The variation in metacentric height due to difference in fill levels inside the water tanks is studied by introducing load cases (LC1-LC8), which were defined based on the damage stability regulation MSC 216(82) SOLAS 2009. The defined load cases shall fall between the loading condition at light ship draft and the loading condition at maximum allowable draft. In the present study, Load Case 1 was a light ship condition, and the corresponding metacentric height was 7.884 m. When the ship was fully loaded (Load Case 8), the corrected metacentric height was found to be 6.107 m. The displacement and draft corresponding to each load case are listed in Table 4. The ship was compartmentalized and divided into 15 zones, as shown in Figure 7.

Table 4. Load Cases

Load Case	Displacement (t)	Draft (m)
1	15886	7.251
2	16245	7.358
3	16518	7.389
4	16964	7.589
5	17324	7.681

6	17686	7.681
7	19121	8.321
8	20918	9.04

The ship was subdivided into compartments, lying in zones as shown in Figure 7a. The compartments at port and starboard side were named and are depicted in Figure 7 (b, c). The zones were assigned to define a damage which may involve a single zone, two adjacent zones, and three or more adjacent zones. A probabilistic damage stability assessment performs analysis on different combinations of zone damages.

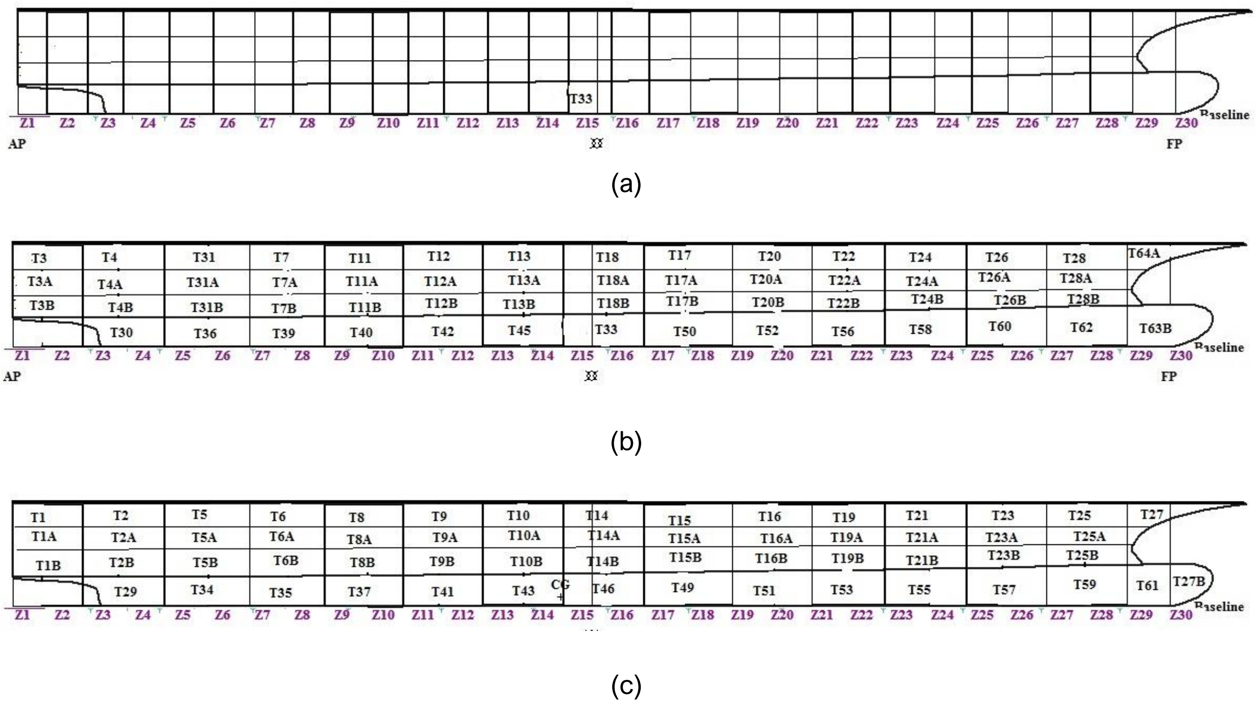


Figure 7. Subdivision a) zone b) compartment-port side c) compartment-starboard side

The MAXSURF damage stability module was used to perform a probabilistic damage stability analysis to obtain the subdivision indices. An attained index of 0.799 was obtained which was more than the required index 0.512. Figure 8 shows the probabilistic damage stability plot. The bottom triangle layer shows the 30 single zone damages and the layers of parallelogram above it show multiple damage locations. The colour bar shows the combinations of damaged compartments (SOLAS chapter II-1 Part B).

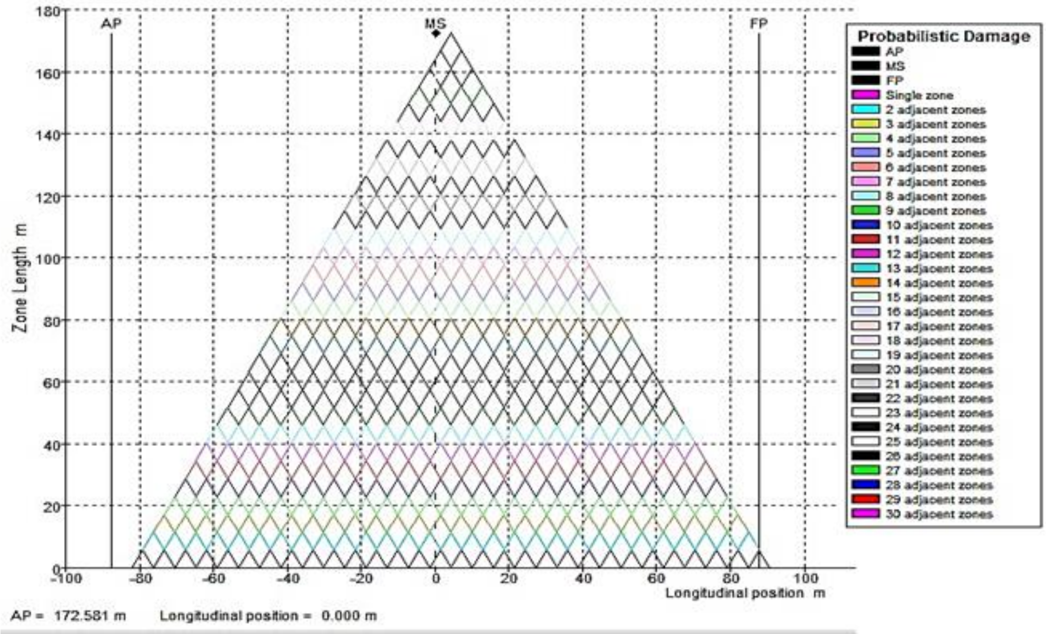


Figure 8. Probabilistic damage stability plot of Ro-Ro ship

As the attained subdivision index was greater than the required subdivision index, the ship was found to be stable under the considered load cases. Thereby the subdivision of the ship provided need not be rearranged to maintain the damaged ship stability. Not all damage cases induced instability, as it is within the floodable length, and will have attained an index greater than required index.

The damage cases (D1 to D22) were defined based on regulation 8 of MSC 216-82. The different compartments were damaged and the permeability of these compartments were defined based on MSC 216-82 (Regulation 7-3, SOLAS 2009), to set the flood level inside compartment. To find the critical damage stability, the shift in metacentric height (GM) for each damage case (D1 to D22) was obtained, as listed below, in Table 5.

Table 5. Shift in metacenter for damage conditions corresponding to load cases

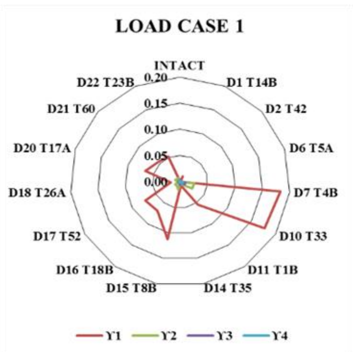
Case	Comp	LC1	LC2	LC3	LC4	LC5	LC6	LC7	LC8
Intact	-	7.021	7.034	7.061	7.023	7.052	7.052	6.778	6.275
D1	T14B	7.00	7.022	7.053	7.026	7.055	7.056	6.786	6.282
D2	T42	7.022	7.118	7.048	7.065	7.10	7.097	6.807	6.308
D3	T11B	7.021	7.034	7.061	7.023	7.052	7.051	6.715	6.187
D4	T31A	7.021	7.034	7.061	7.023	7.052	7.051	6.778	6.275
D5	T6	7.021	7.034	7.061	7.023	7.052	7.051	6.778	6.275
D6	T5A	7.021	7.034	7.061	7.023	7.052	7.051	6.778	6.275
D7	T4B	6.68	6.705	6.809	6.852	6.845	6.815	6.507	6.104
D8	T2A	7.021	7.034	7.061	7.023	7.052	7.051	6.778	6.275
D9	T1	7.021	7.034	7.061	7.023	7.052	7.051	6.778	6.275
D10	T33	7.35	7.297	7.354	7.392	7.409	7.375	7.059	6.563
D11	T1B	6.919	6.884	6.816	6.789	6.717	6.721	6.388	5.927
D12	T12A	7.021	7.034	7.061	7.012	7.03	7.016	6.723	6.218
D13	T13A	7.021	7.034	7.061	7.023	7.052	7.051	6.778	6.275
D14	T35	7.035	7.109	7.156	7.106	7.146	7.129	6.823	6.326
D15	T8B	6.813	6.881	6.94	6.932	6.944	6.931	6.64	6.238
D16	T18B	7.15	7.165	7.178	7.225	7.211	7.204	6.946	6.456

D17	T52	7.157	7.133	7.171	7.198	7.214	7.223	6.987	6.489
D18	T26A	7.053	7.046	6.99	7.025	7.051	7.048	6.821	6.324
D19	T21A	7.157	7.133	7.171	7.198	7.214	7.223	6.987	6.489
D20	T17A	6.897	7.017	6.954	6.997	7.027	7.017	6.762	6.275
D21	T60	7.123	7.039	7.056	7.096	7.083	7.097	6.878	6.377
D22	T23B	7.118	7.078	7.07	7.122	7.156	7.115	6.896	6.398

The damage cases *D3, D4, D5, D6, D8, D9, D12*, D13, and *D19* have the same value of metacentric height as that of intact condition (Table 5). This means that those damage cases do not show much influence on the overall stability of ship when damaged, and it will be within the floodable length. The stability parameter γ is calculated from Eq. 8, for *D1, D2, D7, D10, D11, D14 - D18* and *D20 - D22*. For a set of *α* and *μ* (Table 6), the γ value changes under damaged condition, as the metacentric height in wave (GM_a) and mean metacentric height (GM_m) fluctuate under damaged condition. The stability parameters (γ_{1-4})obtained corresponding to Case 1-4 in Table 6, whereas the load cases and damage cases are shown in radial plots (Figure 9a-h).

Table 6. Calculated values of α, μ

Case:	ω (rad)	α	μ
1	0.20	3.783	0.176
2	0.39	0.536	0.173
3	0.78	0.204	0.143
4	0.98	0.106	0.143



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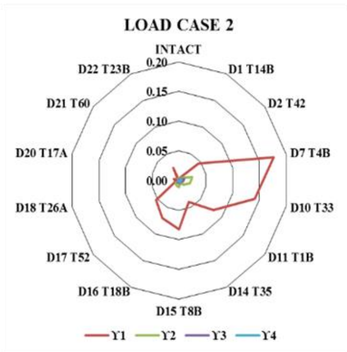

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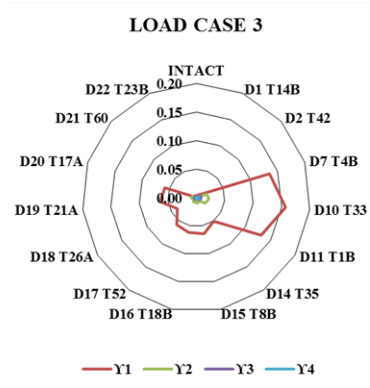

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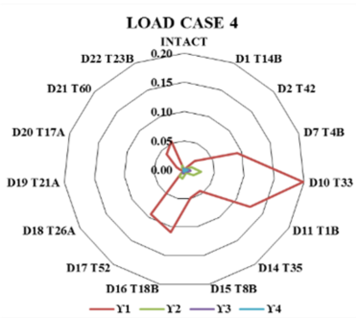

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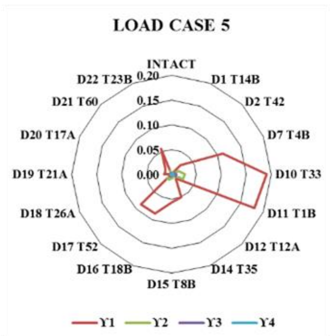
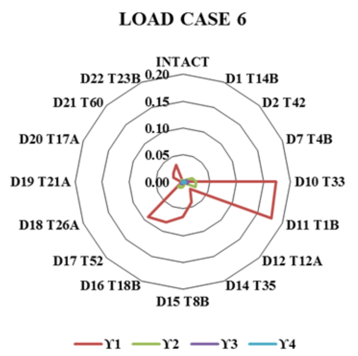


(e)



(f)

(g) (h)

Figure 9 (a-h). Radial plot of stability parameter γ for ship at damaged condition

The stability parameter gamma is dependent on the GM of the ship. There is a change in GM every time the wave encounters the ship. The range depends on the allowable GM variation for the type of vessel under consideration and the operating condition. In all cases, γ_1 value shows greater fluctuation for Case 1, where α =3.78, μ =0.176 and ω =0.2 rad/s. The value of γ_1 is highest (Table 7) when the tanks T4B, T33 and T1B at portside, starboard side, and rear end of the ship are damaged (Figure 10).

Table 7. Maximum value of stability parameter γ_1 for damage cases

Load Case	Damage Case	Tank	γ_1
LC1	D7	T4B	0.18
LC2	D7	T4B	0.18
LC3	D10	T33	0.16
LC4	D10	T33	0.10
LC5	D10	T33	0.19
LC6	D11	T1B	0.18
LC7	D11	T1B	0.22
LC8	D11	T1B	0.22

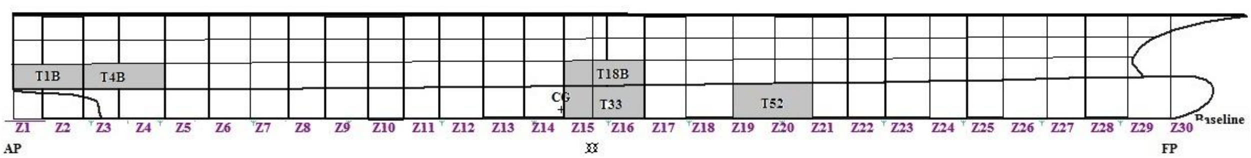


Figure 10. Critical damage location where stability parameter γ is maximum

The capsize behaviour of the damaged ship is discussed in the following section, based on the chaos theory (Senjanovic, 1995).

7. STABILITY BEHAVIOR OF SHIP DURING CAPSIZE

The damaged ship under flooding acts as a nonlinear dynamic system with periodically forced oscillation. The flood water pressure acts as an exciting force for the vessel capsize. The Duffing stability theory includes nonlinear damping and defines the trajectory of convergence and divergence of the chaotic system (extreme instability condition of the ship). In a damaged state, chaotic responses are critical as the flood water dynamics bring forth unexpected instability conditions. The motion equation of ship with floodwater under damaged condition is discussed in Section 3. Considering the ship as a nonlinear dynamic system with periodically forced oscillation, a MATLAB code was developed for the Duffing equation. It helps to understand the chaotic nature of the ship under capsizing condition. The coupled response behaviour of flood water and ship motion was modelled as fifth order polynomial (Senjanovic, 1995), represented as:

$$\ddot{\phi} + [2\xi\omega_0]\dot{\phi} - [(1 - \alpha^2 x^2)(1 - \gamma^2 x^2)\omega_0{}^2]\phi = \mathrm{P.cos}\omega t + \mathrm{U} \tag{15}$$

Where ξ is critical damping, ω_o is natural frequency, $P\cos\omega t$ is excitation force, and U is the additional force due to flooding. By expanding, Eq. 15 takes the following form:

$$\ddot{\phi} + [2\xi\omega_0]\dot{\phi} - \{[(\alpha^2\gamma^2)\omega_0{}^2]x(1)^5 + (\alpha^2 + \gamma^2)\omega_0{}^2]\,x(1)^3 + \omega_0{}^2 x(1)\} = \mathrm{P.cos}\omega t + \mathrm{U} \tag{16}$$

Let, $x(1) = \phi, x(2) = \dot{\phi}$, $\mu = 2\xi\omega_0$

$$F(2) = P\cos wt + U - [2\xi\omega_0]x(2) - \{[(\alpha^2\gamma^2)\omega_0{}^2]x(1)^5 + (\alpha^2 + \gamma^2)\omega_0{}^2]\,x(1)^3 + \omega_0{}^2 x(1)\} \tag{17}$$

Let, $F(1) = \ddot{\phi}, F(2) = \dot{\phi}$

The solution to this equation is obtained from MATLAB as shown below:

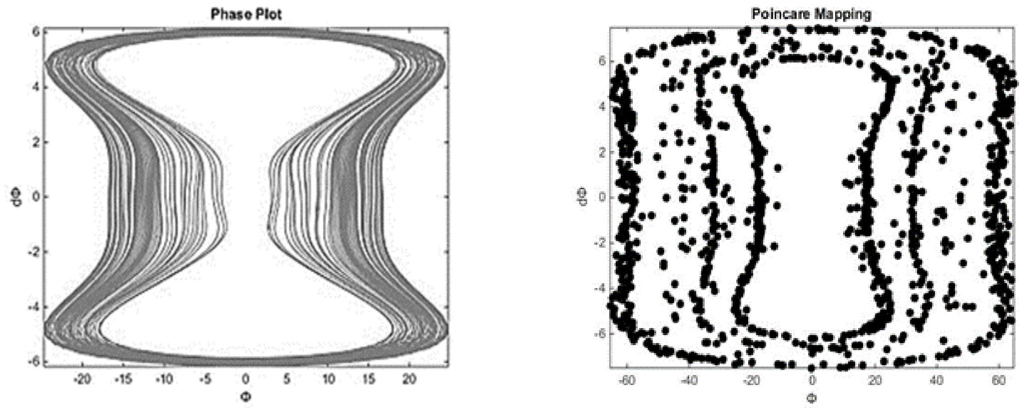
$$F(1) = x(2) \tag{18}$$

$$F(2) = P\cos\omega t + U - \mu.x(2) - \varepsilon x(1)^3 + \psi x(1)^5 \tag{19}$$

In the present study for the Ro-Ro vessel, the external excitation amplitude P=1.5 and the obtained value of damping ξ =0.002 have been considered. Therefore, μ = 0.00136.

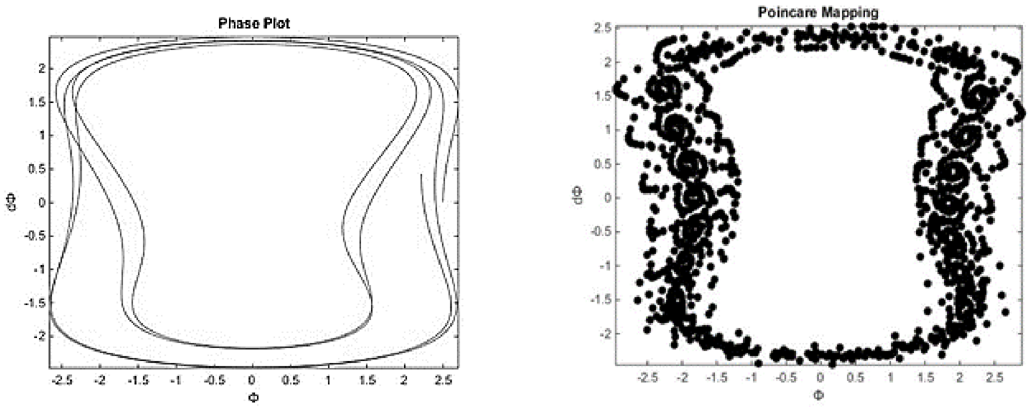
Considering, U =1, P=1.5, α = 3 and γ =1; ε = $\omega_0{}^2$ = 0.1156; ψ = $(\alpha^2+\gamma^2)\omega_0{}^2$ =1.156; ϕ = $(\alpha^2\gamma^2)\omega_0{}^2$ = 1.04.

The varying parameters are ε, ϕ, ψ and the corresponding behaviour of the capsize model is obtained using Poincare mapping and phase plots. The system approaches the chaotic behaviour when the period doubles.The regions of nonlinear resonance are seperated when ε =3.783 ψ =4.239 ϕ = 0.175, where the energy oscillates within the band system (Figure 11). The system attains a stable but vibratory nature at ε =0.1063 ψ =0.07114 ϕ = 0.1436 (Figure 12).



(a) (b)

Figure 11. Energy conserved within the narrow band where ε =3.783 ψ =4.239 ϕ = 0.175 a) Phase diagram b) Poincare mapping (Case 2)



(a) (b)

Figure 12. Vibratory displacement for ε =0.1063 ψ =0.07114 ϕ = 0.1436 a) Phase diagram b) Poincare mapping (Case 4)

8. CONCLUSIONS

A method of obtaining the effect of hydrodynamic coefficients on vessel stability has been formulated. The conditions at which parametric rolling takes place is when the motion frequency is twice the natural frequency. Frequencies at which a vessel becomes stable and unstable were found out for Ro-Ro vessel. This was done based on stability parameters α, γ, μ . A set of eight load cases were considered, along with twenty-two damage cases. Stability parameter variations were tabulated and corresponding γ values obtained. It was found that variation of γ_1 was the maximum in all load cases. A maximum stability variation was observed for damaged tank D7, D10, D11 at starboard, port, and rear end of the ship. At a frequency of 0.2 rad/s, the damaged ship showed a maximum stability variation. It is found that the flooded system exhibits a chaotic nature, similar to that of the Duffing oscillation. The frequencies at which the vibrations occur is captured and the slow shift of excitations with changing frequencies have been observed.

CONFLICT OF INTEREST STATEMENT

The author(s) declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

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